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π-A_1 electromagnetic form factors and light-cone QCD sum rules.

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Abstract

Electromagnetic form factors of the transition $\pi + \gamma_{virt.} \to A_1$ are calculated by QCD sum rules technique with the description of the pion in terms of the set of wave functions of increasing twist. Obtained results are compared with standard QCD sum rule calculations.

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Recently it was suggested to study pion form factor at not very large momentum transfers by light-cone QCD sum rules [1] which combine the description of pion in terms of the set of wave functions of increasing twist with the technique of QCD sum rules [2]. This approach gives a possibility to calculate a contribution of so-called Feynman mechanism to the pion form factor, in which large momentum transfer selects the configuration, in which one parton carries almost all the momentum of the hadron. In that paper [1] it was shown that at least up to the momentum transfers of order 10 GeV^2 this mechanism remains important and it is possible to describe pion form factor in this region by Feynman mechanism only. The second mechanism of large momentum transfer is the hard rescattering mechanism, in which large momentum transfer selects configurations with a small transfers size in which the momentum fraction carried by interacting quark remains an average one. The hard rescattering mechanism involves a hard gluon exchange, and can be written in the factorized form [3], [4], [5].

There exists a number of arguments in favour of that at large but finite momentum transfer ($Q^2 \sim 1 - 10GeV^2$) Feynman mechanism is dominant. Using QCD sum rule approach [2] it was found [6],[7] that pion form factor at $Q^2 \sim 1 - 3GeV^2$ is saturated by Feynman type contribution. However, this method can not be used for higher momentum transfer due to increasing contribution of higher operator product expansion terms at large Q^2. Nevertheless in [8],[9] using a concept of nonlocal condensates it was obtained indications that Feynman type contribution is dominate at least up to $Q^2 \sim 10GeV^2$. At the same time an attempt to describe the data of pion form factor at $Q^2 \geq 3GeV^2$ by the contribution of hard scattering only leads to conclusion that the low energy pion wave function is very far from its asymptotic form [11] and there was proposed a model for pion wave function which has a peculiar "humped" profile which corresponds that the most probable pion configuration is the case when one of the quark carry almost all pion momentum. But in [12] was pointed that the wave function of such type corresponds to respectively soft gluon exchange even at $Q^2 \sim 10GeV^2$.

Here we discuss Feynman type mechanism for $\pi \to A_1$ electromagnetic pion form factors. The first calculation of this amplitude was made in [10] by using operator product expansion for three-points correlator in vacuum.

1

Let us consider the correlator:

$$T_{\mu\nu}(p,q) = \int e^{ipx} d^4x < 0|T\{j^5_\mu(x), j^{em.}_\nu(0)\}|\pi(k) >, \qquad (1)$$

where $j^5_\mu = \bar{d}\gamma_\mu\gamma_5 u$ and $j^{em.}_\nu = \frac{2}{3}\bar{u}\gamma_\nu u - \frac{1}{3}\bar{d}\gamma_\nu d$ is the electromagnetic current, k is momentum of pion. This correlator was used in [1] to study pion form factor.

Leading twist operator gives the following contribution to $T_{\mu\nu}$:

$$T_{\mu\nu}(p,q) = f_\pi \int_0^1 du \frac{\varphi_\pi(u)}{(1-u)p^2 + uq^2} \left(\frac{1}{2}(p^2-q^2)g_{\mu\nu} - 2(1-u)p_\mu p_\nu \right.$$
$$\left. +(1-2u)(p_\mu q_\nu + q_\mu p_\nu) + 2uq_\mu q_\nu\right) \qquad (2)$$

where $\varphi_\pi(u)$ is twist-2 pion wave function. We use the following definition for twist-2 and -4 two-particle wave functions of pion:

$$< 0|\bar{d}(0)\gamma_\mu\gamma_5 u(x)|\pi(k) >= if_\pi k_\mu \int_0^1 e^{-iukx}(\varphi(u) + x^2 g_1(u) + O(x^4))du$$
$$+f_\pi \left(x_\mu - \frac{k_\mu}{kx}x^2\right) \int_0^1 e^{-iukx} g_2(u)du + ... \qquad (3)$$

Here g_1 and g_2 are twist-4 pion wave functions.

In this paper we use different models for pion wave function. The first on is asymptotical wave function. In [3] it was shown that at asymptotically large Q^2 the pion wave function of leading twist has the following form:

$$\varphi^{(as.)}_\pi(u) = 6u(1-u) \qquad (4)$$

The attempt to describe pion form factor at $Q^2 \geq 3GeV^2$ by hard rescattering mechanism only leads to a conclusion that the form of the wave function is much different from asymptotical one and it was suggested to use the following model for pion twist-2 wave function (see [11]):

$$\varphi^{(CZ)}_\pi(u, \mu \sim 500MeV) = 30u(1-u)(2u-1)^2 \qquad (5)$$

And the third model pion wave function is

$$\varphi^{(BF)}_\pi(u) = 6u(1-u)\left(1 + A_2\frac{3}{2}[5(2u-1)^2 - 1]\right.$$
$$\left. +A_4\frac{15}{8}[21(2u-1)^4 - 14(2u-1)^2 + 1]\right) \qquad (6)$$

which was proposed by Braun and Filyanov in [13] at low normalization point $\mu \simeq 0.5 GeV$, with coefficients $A_2 = \frac{2}{3}$ and $A_4 = 0.43$ which is in agreement with QCD sum rules for first moments of pion wave function and provides that at a middle point $u = 0.5$

$$\varphi_\pi(0.5) \simeq 1.2. \tag{7}$$

This value is with experimental values of various hadronic coupling constants calculated be light-cone sum rules approach (see [13]).

It is easy to check that (2) satisfies to Ward Identities:

$$\begin{aligned} p_\mu F_{\mu\nu} &= -f_\pi (p-q)_\nu \\ q_\nu F_{\mu\nu} &= -f_\pi (p-q)_\mu. \end{aligned} \tag{8}$$

Twist-4 quark wave functions $g_1(u)$ and $g_2(u)$ give the following contribution into the correlator (1):

$$\begin{aligned} f_\pi \int_0^1 du \Bigg(& \frac{1}{((1-u)p^2+uq^2)^2} \Big[4g_1(u)(\frac{1}{2}(p^2-q^2)g_{\mu\nu} \\ & -2(1-u)p_\mu p_\nu + (p_\mu q_\nu + q_\mu p_\nu)(1-2u) + 2u q_\mu q_\nu) \\ & -4g_2(u)((1-u)^2 p_\mu p_\nu + u(1-u)(p_\mu q_\nu + q_\mu p_\nu) + u^2 q_\mu q_\nu) \\ & -4\mathcal{G}_2(u)(2(1-u)p_\mu p_\nu - (1-2u)(p_\mu q_\nu + q_\mu p_\nu) - 2u q_\mu q_\nu) \Big] \\ & + \frac{2g_2(u)}{(1-u)p^2+uq^2} g_{\mu\nu} \Bigg), \end{aligned} \tag{9}$$

where $\mathcal{G}_2(u) = -\int_0^u g_2(v)dv$.

There are also quark-gluon twist-4 operators:

$$\begin{aligned} & < 0|\bar{d}(0)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)u(x)|\pi(k) > \\ & = k_\mu(k_\alpha x_\beta - x_\alpha k_\beta)\frac{1}{\pi} f_\pi \int e^{-ikx(\alpha_1+v\alpha_3)} D\alpha \Phi_\parallel(\alpha_i) \\ & + \left(k_\beta(g_{\mu\alpha} - \frac{k_\mu x_\alpha}{kx}) - k_\alpha(g_\mu\beta - \frac{k_\mu x_\beta}{kx}) \right) f_\pi \int e^{-ikx(\alpha_1+v\alpha_3)} D\alpha \Phi_\perp(\alpha_i), \end{aligned} \tag{10}$$

$$\begin{aligned} & < 0|\bar{d}(0)\gamma_\mu i g_s \tilde{G}_{\alpha\beta}(vx)u(x)|\pi(k) > \\ & = k_\mu(k_\alpha x_\beta - x_\alpha k_\beta)\frac{1}{\pi} f_\pi \int e^{-ikx(\alpha_1+v\alpha_3)} D\alpha \Psi_\parallel(\alpha_i) \\ & + \left(k_\beta(g_{\mu\alpha} - \frac{k_\mu x_\alpha}{kx}) - k_\alpha(g_\mu\beta - \frac{k_\mu x_\beta}{kx}) \right) f_\pi \int e^{-ikx(\alpha_1+v\alpha_3)} D\alpha_i \Psi_\perp(\alpha_i), \end{aligned} \tag{11}$$

where $\tilde{G}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}G^{\mu\nu}$, $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$.

The contribution of these operators (10,11) is:

$$f_\pi \int_0^1 du \frac{1}{((1-u)p^2+uq^2)^2}\left[(2(1-u)p_\mu p_\nu - (1-2u)(p_\mu q_\nu + q_\mu p_\nu)\right.$$
$$\left. -2uq_\mu q_\nu)A(u) - \frac{1}{2}g_{\mu\nu}(p^2-q^2)B(u) + 2(p_\mu q_\nu - q_\mu p_\nu)C(u)\right], \quad (12)$$

where

$$A(u) = \int_0^u d\alpha_1 \int_{u-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3}\left[\Psi_\parallel(\alpha_i) + 2\Psi_\perp(\alpha_i)\right.$$
$$\left. + \left(1 - 2\frac{u-\alpha_1}{\alpha_3}\right)(\Phi_\parallel(\alpha_i) + 2\Phi_\perp(\alpha_i))\right], \quad (13)$$

$$B(u) = \int_0^u d\alpha_1 \int_{u-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3}\left[\Psi_\parallel(\alpha_i) + \left(1-2\frac{u-\alpha_1}{\alpha_3}\right)\Phi_\parallel(\alpha_i)\right], \quad (14)$$

$$C(u) = \int_0^u d\alpha_1 \int_{u-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3}\left[\Phi_\perp(\alpha_i) + \left(1-2\frac{u-\alpha_1}{\alpha_3}\right)\Psi_\perp(\alpha_i)\right]. \quad (15)$$

There are also 4-particle twist-4 operators which were considered in ref.[14]. Contributions of such operators are not considered here.

A systematic study of the higher twist wave functions (excluding four-particle operators) has been done in the paper [13]. The set of wave functions suggested in the paper includes contributions of operators with lowest conformal spin and also the corrections corresponding to the operators with next-to-leading conformal spin, which numerical values were estimated by the QCD sum rule method. This set is (hereafter $\bar{u} = 1-u$):

$$\begin{aligned}
\Phi_\parallel(\alpha_i) &= 120\varepsilon\delta^2(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3, \\
\Psi_\parallel(\alpha_i) &= -120\delta^2\alpha_1\alpha_2\alpha_3\left(\frac{1}{3}+\varepsilon(1-3\alpha_3)\right), \\
\Phi_\perp(\alpha_i) &= 30\delta^2(\alpha_1-\alpha_2)\alpha_3^2\left(\frac{1}{3}+2\varepsilon(1-2\alpha_3)\right), \\
\Psi_\perp(\alpha_i) &= 30\delta^2(1-\alpha_3)\alpha_3^2\left(\frac{1}{3}+2\varepsilon(1-2\alpha_3)\right),
\end{aligned} \quad (16)$$

$$\begin{aligned}
g_1(u) &= \frac{5}{2}\delta^2\bar{u}^2u^2 + \frac{1}{\varepsilon}\delta^2\left(\bar{u}u(2+13\bar{u}u) + 10u^3(2-3u+\frac{6}{5}u^2)\ln(u)\right.\\
&\left.+10\bar{u}^3(2-3\bar{u}+\frac{6}{5}\bar{u}^2)\ln(\bar{u})\right),\\
g_2(u) &= \frac{10}{3}\delta^2\bar{u}u(u-\bar{u}),\\
\mathcal{G}_2(u) &= \frac{5}{3}\delta^2\bar{u}^2u^2, \qquad (17)
\end{aligned}$$

where

$$\delta^2 \simeq 0.2GeV^2, \quad \varepsilon \simeq 0.5. \qquad (18)$$

The value of δ^2 was determined in [15].

Using this set of the wave function we obtain the following expressions for $A(u)$, $B(u)$ and $C(u)$:

$$\begin{aligned}
A(u) &= \delta^2\left(10u^2(1-u)^2 + \varepsilon\left[-4u - 22u^2 + 52u^3 - 26u^4 - 4\ln(1-u)\right.\right.\\
&\left.\left.+4u^3(10-15u+12u^2)\ln\left(\frac{1-u}{u}\right)\right]\right), \qquad (19)\\
B(u) &= \delta^2\left(-10u^2(1-u)^2 + \varepsilon\left[-4u - 22u^2 + 52u^3 - 26u^4 - 4\ln(1-u)\right.\right.\\
&\left.\left.+4u^3(10-15u+12u^2)\ln\left(\frac{1-u}{u}\right)\right]\right), \qquad (20)\\
C(u) &= 20\delta^2\varepsilon u^2(1-u)^2(2u-1). \qquad (21)
\end{aligned}$$

2

Let us consider hadron contribution into correlator $T_{\mu\nu}$. A_1 -meson gives the following contribution into this correlator:

$$\begin{aligned}
&\frac{-i}{m_A^2 - p^2} < 0|j_\mu^5|A_1(p)>< A_1(p)|j_\nu^{em.}|\pi(k)>\\
&= \frac{f_A}{m_A^2 - p^2}\left[g_{\mu\nu}m_A^2 G_1(Q^2) - p_\mu p_\nu\frac{Q^2}{m_A^2}\left(G_1(Q^2) - \left(1 - \frac{Q^2}{m_A^2}\right)G_2(Q^2)\right)\right.\\
&\quad -p_\mu q_\nu\frac{1}{2}\left(1-\frac{Q^2}{m_A^2}\right)\left(G_1(Q^2) - \left(1-\frac{Q^2}{m_A^2}\right)G_2(Q^2)\right)
\end{aligned}$$

$$-2q_\mu p_\nu \left(G_1(Q^2) + \frac{Q^2}{m_A^2} G_2(Q^2) \right)$$
$$+q_\mu q_\nu \left(G_1(Q^2) - \left(1 - \frac{Q^2}{m_A^2}\right) G_2(Q^2) \right) \Bigg], \quad (22)$$

where

$$< 0|j^5_\mu|A_1(p) >= i\epsilon_\mu m_A f_A, \quad (23)$$

$$< A_1|j^{em.}_\nu|\pi(k) >= -\frac{\epsilon_\lambda}{m_A} \Big[(2p-q,q)g_{\nu\lambda} - (2p-q)_\nu q_\lambda) G_1(Q^2)$$
$$-\frac{1}{m_A^2}((2p-q,q)q_\nu - q^2(2p-q)_\nu)q_\lambda G_2(Q^2) \Big], \quad (24)$$

$m_A = 1.26 GeV$ is A_1-meson mass, ϵ_λ is a polarization of the meson, $f_A = 0.2$ which was determined in [16] from a fit to the absolute rate for $\tau \to \nu_\tau \pi\pi\pi$ decay. Notice that in paper [10] it was used another set for mass and A_1-coupling constant - $\frac{g^2_{A_1}}{4\pi} \simeq 6.0$ and $m_A = 1.15 GeV$. This parameters were determined in [17] by QCD sum rule, $f_A = \sqrt{2}\frac{m_A}{g_{A_1}}$.

The heavier spin-1 states contributions have the same form as (22).

In the limit of massless quarks due to the conservation of axial current j^5_μ, the only massless state of spin-0 (pion) gives nonvanishing contribution into the correlator (1). This contribution is

$$\frac{i}{p^2} < 0|j^5_\mu|\pi(p) >< \pi(p)|j^{em.}_\nu|\pi(k) >= -\frac{f_\pi p_\mu}{p^2} F_\pi(Q^2)(2p-q)_\nu. \quad (25)$$

Pion form factor $F_\pi(Q^)$ was studied in [1].

3

To study G_1 form factor we consider correlation function $f_1(p^2, q^2)$:

$$T_{\mu\nu}(p,q) = g_{\mu\nu} f_1(p^2,q^2) + ...(other \;\; tensors). \quad (26)$$

Then according to (14) and (25) it is possible to write the following dispersional relation for the structure $g_{\mu\nu}$:

$$f_1(p^2,q^2) = \frac{f_A m_A^2 G_1(Q^2)}{m_A^2 - p^2} + \frac{1}{\pi}\int_{s_0}^\infty \frac{\rho(s,Q^2)ds}{p^2 - s} + (subtraction\, terms), \quad (27)$$

where $\rho(s,Q^2)$ is the spectral density of higher state (spin-1) contribution into the correlator (1), $s_0 > m_A^2$ and pion contribution is absent.

To suppress higher states contribution and to kill subtraction terms in (27) we use so called Borel operator $\mathcal{B}$ which was suggested in [2]:

$$\mathcal{B}_{P^2} f(P^2) = \frac{(P^2)^{(n+1)}}{n!}\left(-\frac{d}{dP^2}\right)^n f(P^2) = f^B(t),$$
$$\frac{P^2}{t} = n, \quad P^2 = -p^2, \quad n \to \infty. \tag{28}$$

Applying this operator to (27) we obtain

$$f_1^B(t,Q^2) = f_A m_A^2 G_1(Q^2) e^{-\frac{m_A^2}{t}} + \frac{1}{\pi}\int_{s_0}^{\infty} e^{-\frac{s}{t}}\rho(s,Q^2)ds. \tag{29}$$

Notice, that higher states contribution is suppressed by a factor $e^{-s/t} \leq e^{-s_0/t}$.

From other side, the leading twist-2 wave function gives the following contribution to f_1^B:

$$f_1^{B(twist-2)} = \frac{f_\pi}{2}\int_0^1 \frac{Q^2\varphi_\pi(u)}{(1-u)^2} e^{-\frac{uQ^2}{(1-u)t}} du$$
$$= \frac{f_\pi}{2}\int_0^{\infty} \varphi_\pi\left(\frac{s}{s+Q^2}\right) e^{-\frac{s}{t}} ds. \tag{30}$$

In (30) we represent the formula in the form of dispersional integral after Borel transformation and $\frac{f_\pi}{2}\varphi_\pi\left(\frac{s}{s+Q^2}\right)$ is equal to imaginary part of the correlator at $p^2 = s$.

According to (9) and (12) twist-4 wave functions contributions are

$$f_1^{B(twist-4)} = \frac{f_\pi}{2}\int_0^1 \left(\frac{Q^2(4g_1(u)-B(u))}{t(1-u)^3} - \frac{4g_1(u)-B(u)}{(1-u)^2}\right.$$
$$\left.-\frac{4g_2(u)}{(1-u)}\right) e^{-\frac{uQ^2}{(1-u)t}} du = \frac{f_\pi}{2}\int_0^{\infty} ds \left(\left[4g_1\left(\frac{s}{s+Q^2}\right)\right.\right.$$
$$\left.\left.-B\left(\frac{s}{s+Q^2}\right)\right]\left[\frac{s+Q^2}{Q^2 t} - \frac{1}{Q^2}\right] - \frac{4g_2\left(\frac{s}{s+Q^2}\right)}{s+Q^2}\right) e^{-\frac{s}{t}} ds. \tag{31}$$

Due to asymptotic freedom in the limit $s \to \infty$ the imaginary part of the correlator tends to his perturbative value. So it is possible, like in usual sum rules, to estimate higher states contribution suggested that the imaginary part of the correlator (1) is equal to its calculable part (by operator product expansion) starting at $s = s_0$. Or by other words, here we use the standard concept of duality, which tell us that A_1-meson occupies the "region of duality" in the invariant mass s up to a certain threshold $s_0 \simeq 3GeV^2$. Thus, to take into account higher states contribution we use the following limits in of integration (30,31):

$$0 \leq u \leq \frac{s_0}{s_0 + Q^2} \ \ or \ \ 0 \leq s \leq s_0. \tag{32}$$

Using eqs.(29-32) we obtain the following sum rule:

$$G_1(Q^2) = \frac{f_1^{B(twist-2)} + f_1^{(B(twist-4)}}{f_A m_A^2} e^{\frac{m_A^2}{t}}, \tag{33}$$

where we use a standard model for higher state contribution using the limits of integration (32) in expressions for $f_1^{B(2)}$ and $f_1^{B(4)}$.

Three models of the leading twist wave function $\varphi_\pi(u)$ were considered. In Fig.1 it was shown Q^2-dependence of G_1 form factor which was calculated using eq.(33) at $t = 1.5GeV^2$ and $s_0 = 3GeV^2$.

Notice, that the dependence of G_1 on the pion wave function is very weak at $Q^2 \sim 2.5GeV^2$. The reason of this is that at $Q^2 \sim s_0 = 3GeV^2$ we integrate a pion wave function in the region $0 < u < 0.5$. In the limit $t \to \infty$ this integral is equal to 0.5 and does not depend on the form of wave function. It is clear that using for respectively large t the dependence of sum rule on the pion wave function will be weak and at $Q^2 \simeq 2.5GeV^2$ this dependence is compensated by a small changing of integration region over u.

In this picture we show predictions which were obtained by Ioffe and Smilga in [10] from QCD sum rules for three-point correlator. Notice that there is a big disagreement between predictions of QCD sum rules for three-point correlator and the case of asymptotic wave function for π-meson. The best agreement with predictions of Ioffe and Smilga at $Q^2 \leq 2GeV^2$ we have in the case of BF-wave function. At $Q^2 \sim 3GeV^2$ the predictions of [10] two times smaller than the result obtained in this paper. Probably, this disagreement can be explained by large contribution of higher dimensions

operators which becomes large at this momentum transfer (see details in [10]). At $Q^2 \leq 2GeV^2$ disagreement between the case of BF-wave function and three-point sum rule is about 20% which is usual accuracy of QCD sum rules. In this picture we show the experimental value for $G_1(0)$ which is determined from partial width of decay $A_1 \to \pi\gamma$. According to [18] $\Gamma(A_1 \to \pi\gamma) = 640 \pm 246 keV$. Stability of the sum rule (33) is illustrated in Fig.2. Notice that at $Q^2 \geq 15GeV^2$ continuum contribution becomes larger than 30%. Twist-4 operator contribution is smaller than 10%.

4

To find G_2 form factor, let us consider $q_\mu p_\nu$ and $q_\mu q_\nu$ tensor structures of contribution of hadrons into the correlator (1). The only spin-1 states give nonzero contribution into these structures. In the case $q_\mu p_\nu$ this contribution has the following form:

$$f_\alpha(p^2,Q^2) = -\sum_i \frac{2f_i}{m_i^2-p^2}\left(G_1^{(i)}(Q^2) + \frac{Q^2}{m_i^2}G_2^{(i)}(Q^2)\right), \tag{34}$$

and in the case $q_\mu q_\nu$ the contribution is:

$$f_\beta(p^2,Q^2) = \sum_i \frac{f_i}{m_i^2-p^2}\left(\left(G_1^{(i)}(Q^2) + \frac{Q^2}{m_i^2}G_2^{(i)}(Q^2)\right) - G_2^{(i)}(Q^2)\right), \tag{35}$$

where $\sum_i$ is a sum over spin-1 resonances (A_1 and higher states with $J^{PC} = 1^{+-}$, m_i and f_i are their masses and residues into axial current, $G_1^{(i)}$ and $G_2^{(i)}$ are their form factors. From other side, f_α and f_β were calculated in Section II. Thus using eqs.(34,35) we have the following sum rule for G_2 form factor:

$$-\frac{1}{2}f_\alpha(p^2,Q^2) - f_\beta(p^2,Q^2) = \sum_i \frac{f_i}{m_i^2-p^2}G_1^{(i)}(Q^2). \tag{36}$$

The left side of this expression was found in Section II at $p^2 \sim 1-3GeV^2$ and $Q^2 > 1GeV^2$ Using Borel operator (see (28)) and using the model of continuum which was described in previous Section to take into account higher state contributions we obtain the following sum rule:

$$\frac{f_\pi}{f_A}e^{\frac{m_A^2}{t}}\int_0^{u_0} e^{-\frac{Q^2u}{t(1-u)}}du\left\{\frac{(\frac{1}{2}+u)\varphi_\pi(u)}{1-u}\right.$$

$$+\frac{1}{(1-u)^2t}\left((1+2u)(-2g_1(u)-2\mathcal{G}_2(u)\right.$$
$$\left.\left.+\frac{1}{2}A(u))+2u(1+u)g_2(u)+C(u)\right)\right\} \tag{37}$$

$$= \frac{f_\pi}{f_A}e^{\frac{m_A^2}{t}}\int_0^{s_0}\left\{\frac{(3s+Q^2)\varphi_\pi\left(\frac{s}{s+Q^2}\right)}{2(s+Q^2)^2}\right.$$
$$+\frac{1}{Q^2t}\left[\frac{3s+Q^2}{s+Q^2}\left(-2g_1\left(\frac{s}{s+Q^2}\right)-2\mathcal{G}_2\left(\frac{s}{s+Q^2}\right)+\frac{1}{2}A\left(\frac{s}{s+Q^2}\right)\right)\right.$$
$$\left.\left.+2\frac{s(2s+Q^2)}{(s+Q^2)^2}g_2\left(\frac{s}{s+Q^2}\right)+C\left(\frac{s}{s+Q^2}\right)\right]\right\}. = G_2(Q^2) \tag{38}$$

Q^2 dependence of G_2 form factor for different wave functions are shown in Fig.3. These results were obtained from sum rule (38) at $t = 1.5GeV^2$ and $s_0 = 3GeV^2$. It is interesting compare the results obtained with the first calculation of this formfactor which was made by Ioffe and Smilga [10]. They used QCD sum rules for three point correlator and their results are depicted in Fig.3 too. In the case of this form factor there is a good agreement between the case of asymptotic wave function and predictions which was obtained in [10]. We have a big disagreement between our results for CZ and BF wave functions at $Q^2 \sim 1GeV^2$. This disagreement can be explained by a large contribution of higher twist operators. At $Q^2 > 2GeV^2$ the contribution of twist-4 operators becomes smaller 20%. At lower $Q^2 \sim 1GeV^2$ this contribution is more than 30%. Stablity of the sum rules is illustrated in Fig.4.

5

In this paper we have used light-cone sum rules to calculate electromagnetic $\pi - A_1$ form factors in the region of intermediate momentum transfers: $1GeV^2 < Q^2 < 15GeV^2$. It was studied the dependence of the results obtained on the form of the pion wave function of leading twist. It was found that at $Q^2 > 3GeV^2$ the behaviour of the form factors in the case of asymptotical wave function are very different from the cases of CZ and BF wave functions. It was shown that the results obtained in the cases of CZ and BF wave functions are not very different from each other at $Q^2 \sim 10GeV^2$.

We compare the results obtained in this paper with the first calculations of these form factors which was made in framework of QCD sum rules for three point correlator by Ioffe and Smilga [10]. These two predictions are in agreement within 20-30% accuracy at $Q^2 \sim 1-2GeV^2$. This accuracy is usual accuracy of QCD sum rules. It is shown that there are disagreements between these two approaches at $Q^2 > 2GeV^2$ and $Q^2 < 1GeV^2$ which can be explained by large contribution of power corrections at high Q^2 in the case of three-points sum rule and by contribution of higher twist operators at respectively small Q^2 in the case of light-cone sum rules.

We obtain that vector dominance does not work in the process. If we try to fit the form factors by ρ and ρ'-mesons then we find that ρ'-meson contribution is dominant at $Q^2 > 1GeV^2$, and we can not use vector dominance to estimate $\rho - A_1 - \pi$-coupling constant.

It is shown that at $Q^2 > 1-2GeV^2$ form factor G_1 increases with growth of Q^2. This behaviour can be explained in framework of naive quark model, where A_1-meson is consist of two quarks in P-wave.

Notice that it will be very interesting to measure G_1 form factor at $Q^2 \sim 2.5GeV^2$ because here we have a very weak dependence on the form of pion wave function. The slope of the form factor gives additional information on the function.

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Appendix A

In [13] it was shown that there are relations between wave functions of the same twist. Here one more relation for twist-4 wave function will be obtain. Notice, that a set of wave functions suggested in [13] is in agreement with this relation. It is a generalization of one of the relations obtained in [13].

Let us consider the following matrix element:

$$\frac{1}{f_\pi} < 0|\bar{d}(0)\gamma_\mu\gamma_\rho\gamma_\nu\gamma_5 u(x)|\pi >= (g_{\mu\rho}q_\nu + g_{\nu\rho}q_\mu - g_{\mu\nu}q_\rho)$$
$$\times \int_0^1 due^{-iuqx}\left(i\varphi_\pi(u) + ix^2 g_1(u) - \frac{x^2}{(qx)}g_2(u)\right)$$

$$+(g_{\mu\rho}x_\nu + g_{\nu\rho}x_\mu - g_{\mu\nu}x_\rho)\int_0^1 due^{-iuqx}g_2(u), \tag{A1}$$

and find first derivative of (A1) over x_ν

$$\begin{aligned} &ig_{\mu\rho}(qx)\int_0^1 due^{-iuqx}(2g_1(u)+2\mathcal{G}_2(u)-ug_2(u)) \\ &+i(q_\mu x_\rho - q_\rho x_\mu)\int_0^1 due^{-iuqx}(2g_1(u)+2\mathcal{G}_2(u)+ug_2(u)). \end{aligned} \tag{A2}$$

From other side, using equation of motion for massless quarks we have:

$$\begin{aligned} &\frac{1}{f_\pi}\frac{\partial}{\partial x_\nu} < 0|\bar{d}(0)\gamma_\mu\gamma_\rho\gamma_\nu\gamma_5 u(x)|\pi > \\ &= i\frac{1}{f_\pi}\int_0^1 vdv < 0|\bar{d}(0)\gamma_\mu\gamma_\rho\gamma_\nu\gamma_5 x_\alpha gG_{\alpha\nu}(vx)u(x)|\pi > \end{aligned} \tag{A3}$$

$$\begin{aligned} &= i\frac{1}{f_\pi}\int_0^1 vdvx_\alpha\left((g_{\mu\rho}g_{\nu\lambda}+g_{\nu\rho}g_{\mu\lambda}-g_{\mu\nu}g_{\rho\lambda})\right. \\ &\left.< 0|\bar{d}(0)gG_{\alpha\nu}(vx)\gamma_\lambda\gamma_5 u(x)|\pi > +\varepsilon_{\mu\nu\rho\lambda} < 0|\bar{d}(0)igG_{\alpha\nu}\gamma_\lambda u(x)|\pi >\right) \end{aligned} \tag{A4}$$

$$\begin{aligned} &= ig_{\mu\rho}(qx)\int_0^1 vdv\mathcal{D}\alpha_i e^{-iqx(\alpha_1+v\alpha_3)}\left(\Phi_\parallel(\alpha_i)-2\Phi_\perp(\alpha_i)\right) \\ &+i(q_\mu x_\rho - x_\mu q_\rho)\int_0^1 vdv\mathcal{D}\alpha_i e^{-iqx(\alpha_1+v\alpha_3)}\left(\Phi_\parallel(\alpha_i)+2\Psi_\perp(\alpha_i)\right). \end{aligned} \tag{A5}$$

Using new variable u:

$$\alpha_1 + v\alpha_3 = u \quad v = \frac{u-\alpha_1}{\alpha_3} \tag{A6}$$

eq.(A5) becomes

$$ig_{\mu\rho}(qx)\int_0^1 e^{-iqxu}du\mathcal{A}(u) + i(q_\mu x_\rho - x_\mu q_\rho)\int_0^1 e^{-iqxu}du\mathcal{B}(\sqcap), \tag{A7}$$

where

$$\mathcal{A}(u) = \int_0^u d\alpha_1\int_{u-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3^2}(u-\alpha_1)\left(\Phi_\parallel(\alpha_i)-2\Phi_\perp(\alpha_i)\right), \tag{A8}$$

$$\mathcal{B}(u) = \int_0^u d\alpha_1\int_{u-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3^2}(u-\alpha_1)\left(\Phi_\parallel(\alpha_i)+2\Psi_\perp(\alpha_i)\right). \tag{A9}$$

Comparing eqs.(A2) and (A7) we obtain:

$$\begin{aligned} 2g_1(u) &- 2\mathcal{G}_2(u) - ug_2(u) \\ &= \int_0^u d\alpha_1 \int_{u-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3^2}(u-\alpha_1)\left(\Phi_\parallel(\alpha_i) - 2\Phi_\perp(\alpha_i)\right), \end{aligned} \quad \text{(A10)}$$

$$\begin{aligned} 2g_1(u) &+ 2\mathcal{G}_2(u) + ug_2(u) \\ &= \int_0^u d\alpha_1 \int_{u-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3^2}(u-\alpha_1)\left(\Phi_\parallel(\alpha_i) + 2\Psi_\perp(\alpha_i)\right). \end{aligned} \quad \text{(A11)}$$

The first relation (A10) was obtained in [13]. The second equation (A11) is a new one. Using (A10) and (A11) we obtain

$$2\mathcal{G}_2(u) + ug_2(u) = \int_0^u d\alpha_1 \int_{u-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3^2}(u-\alpha_1)\left(\Phi_\perp(\alpha_i) + \Psi_\perp(\alpha_i)\right). \quad \text{(A12)}$$

It is not difficult to check that the set of wave functions suggested in [13] satisfies to this new relation (A12). Notice that the set of wave functions was fixed by using two integral relations which are valid for any twist-4 wave functions, and additional one which was obtained for asymptotic and preasymptotic twist-4 wave functions. Thus, this new integral relation is a generalization of the constraints obtained in [13] for the case of asymptotic and preasymptotic wave functions.

Let we write the last relation for twist-4 wave function which was obtained in [13]:

$$g_2(u) = \int_0^u d\alpha_1 \int_{u-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3}\left(2\Phi_\perp(\alpha_i) - \Phi_\parallel(\alpha_i)\right) \quad \text{(A13)}$$

Equation (A12) can be useful for construction of a set of wave functions which are far from asymptotic ones.

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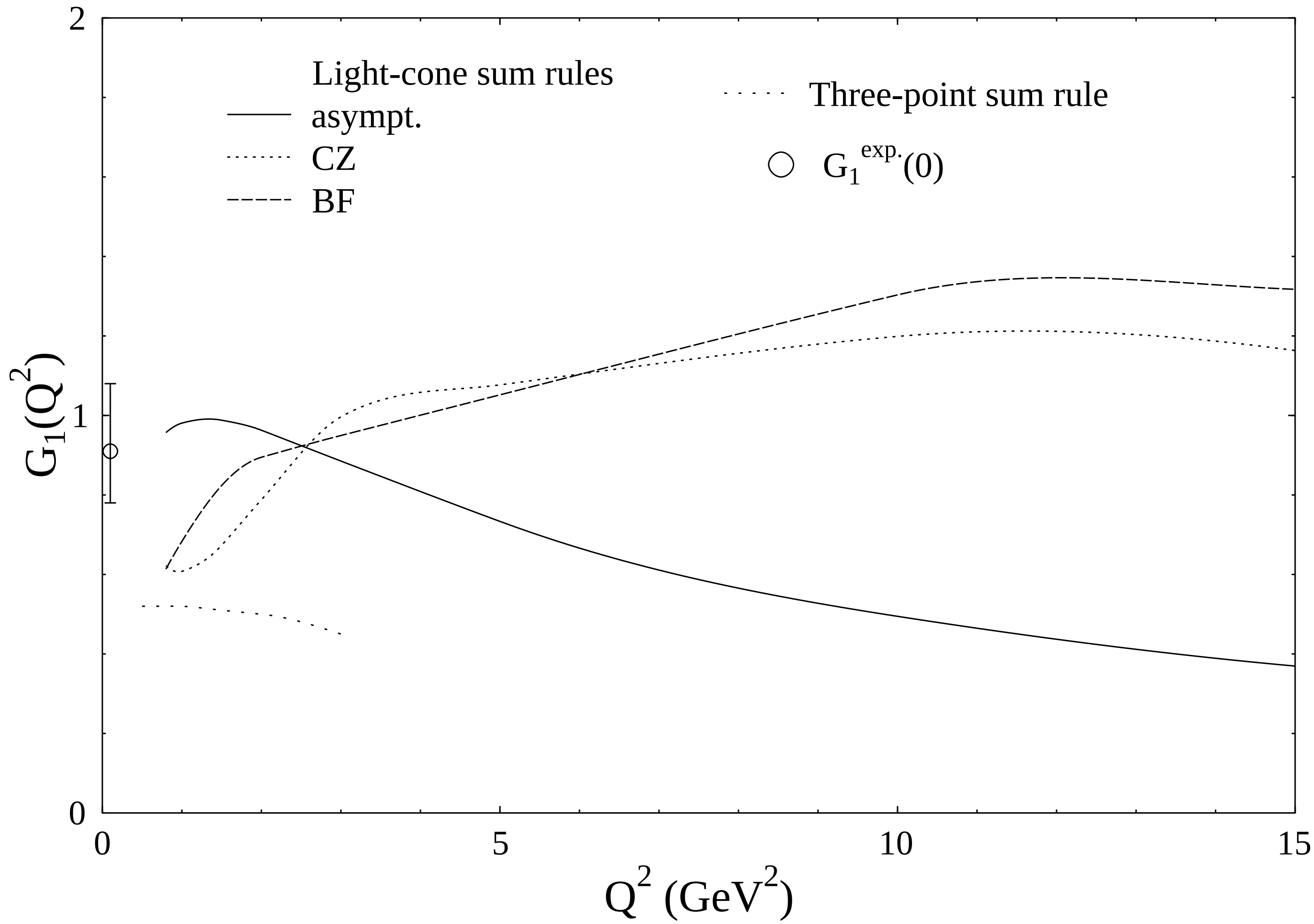

Fig.1. Form Factor $G_1(Q^2)$.

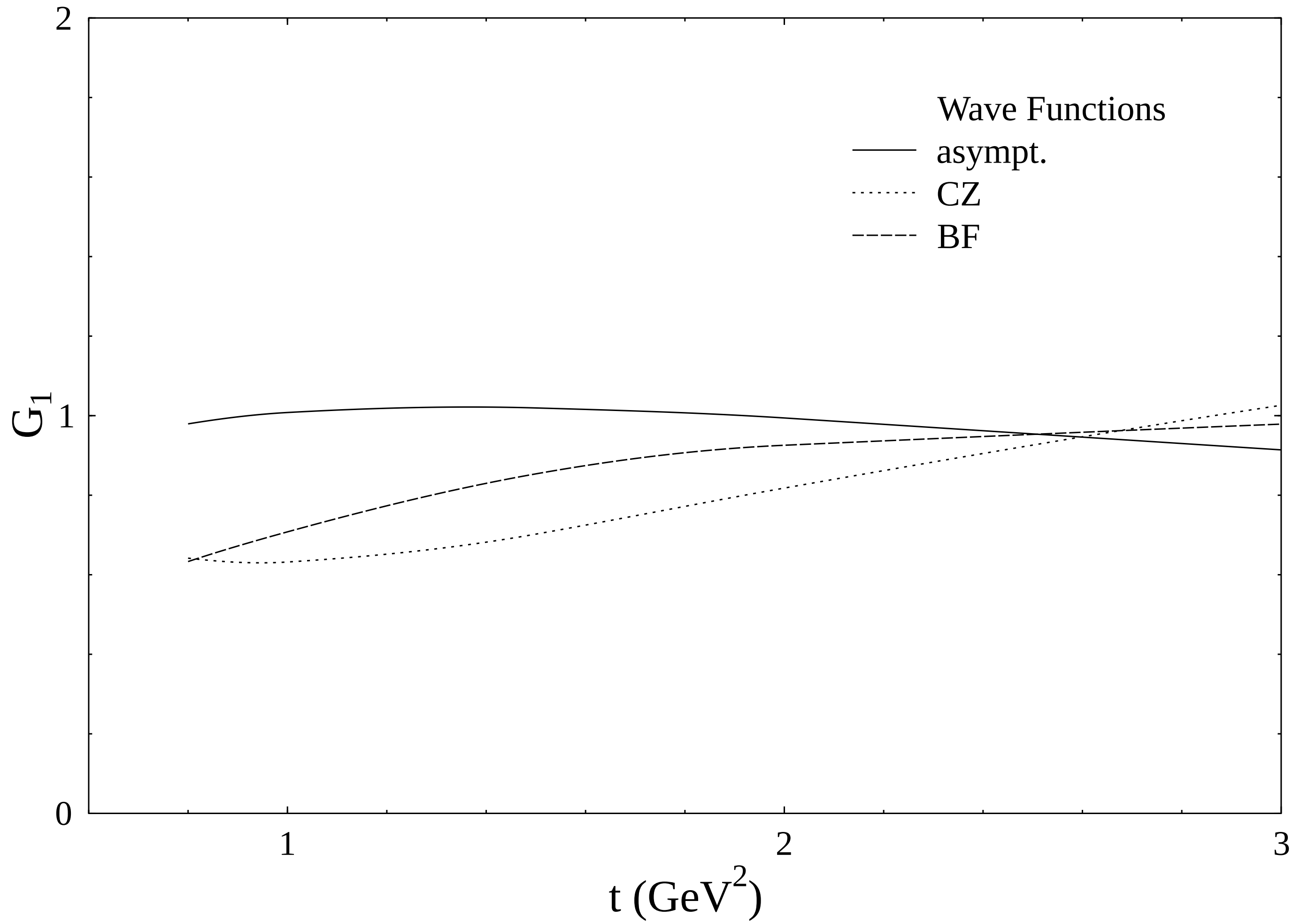

Fig.2. t-dependence of the sum rule for $G_1(Q^2)$, Q^2=2GeV^2.

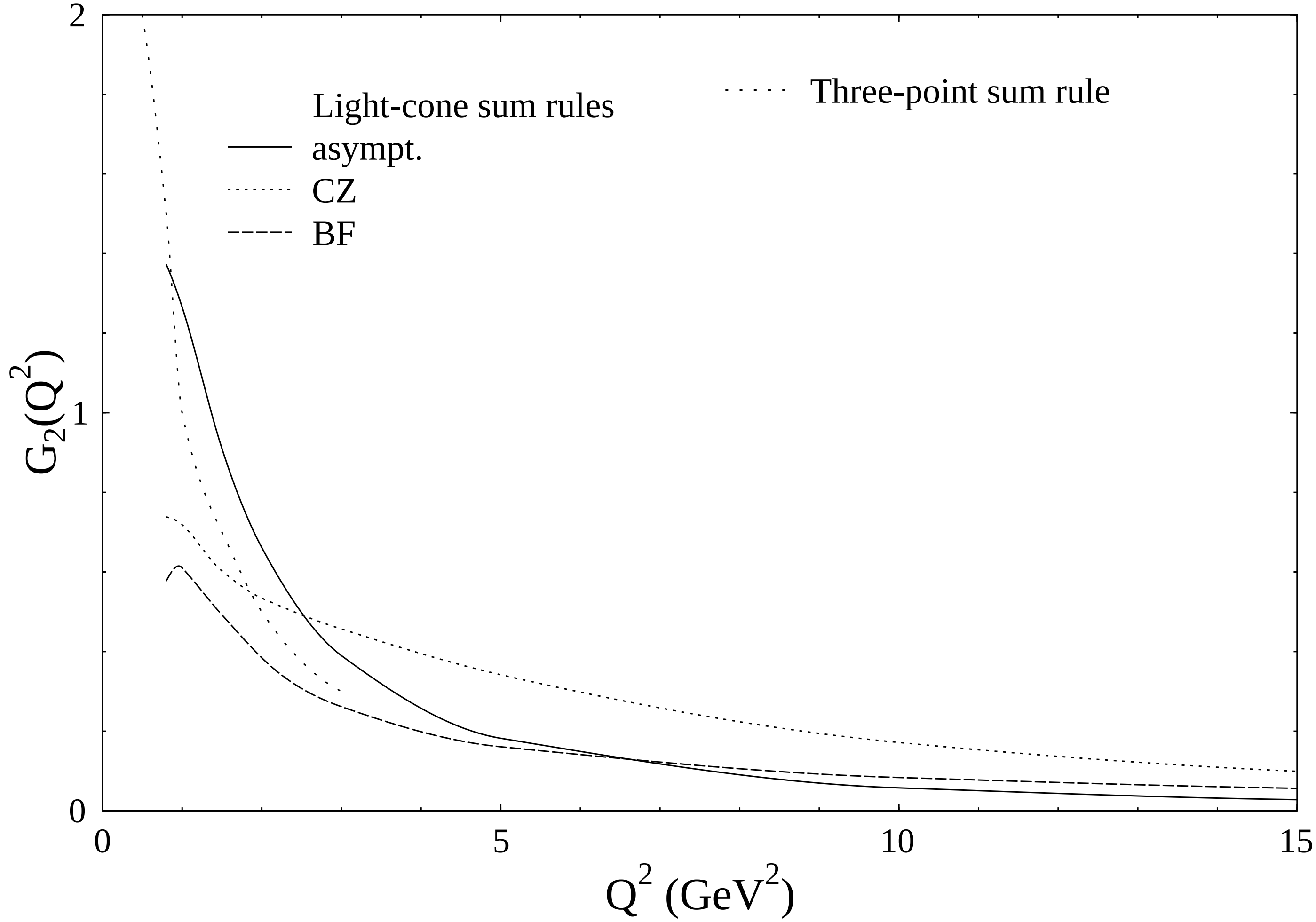

Fig.3. Form Factor $G_2(Q^2)$.

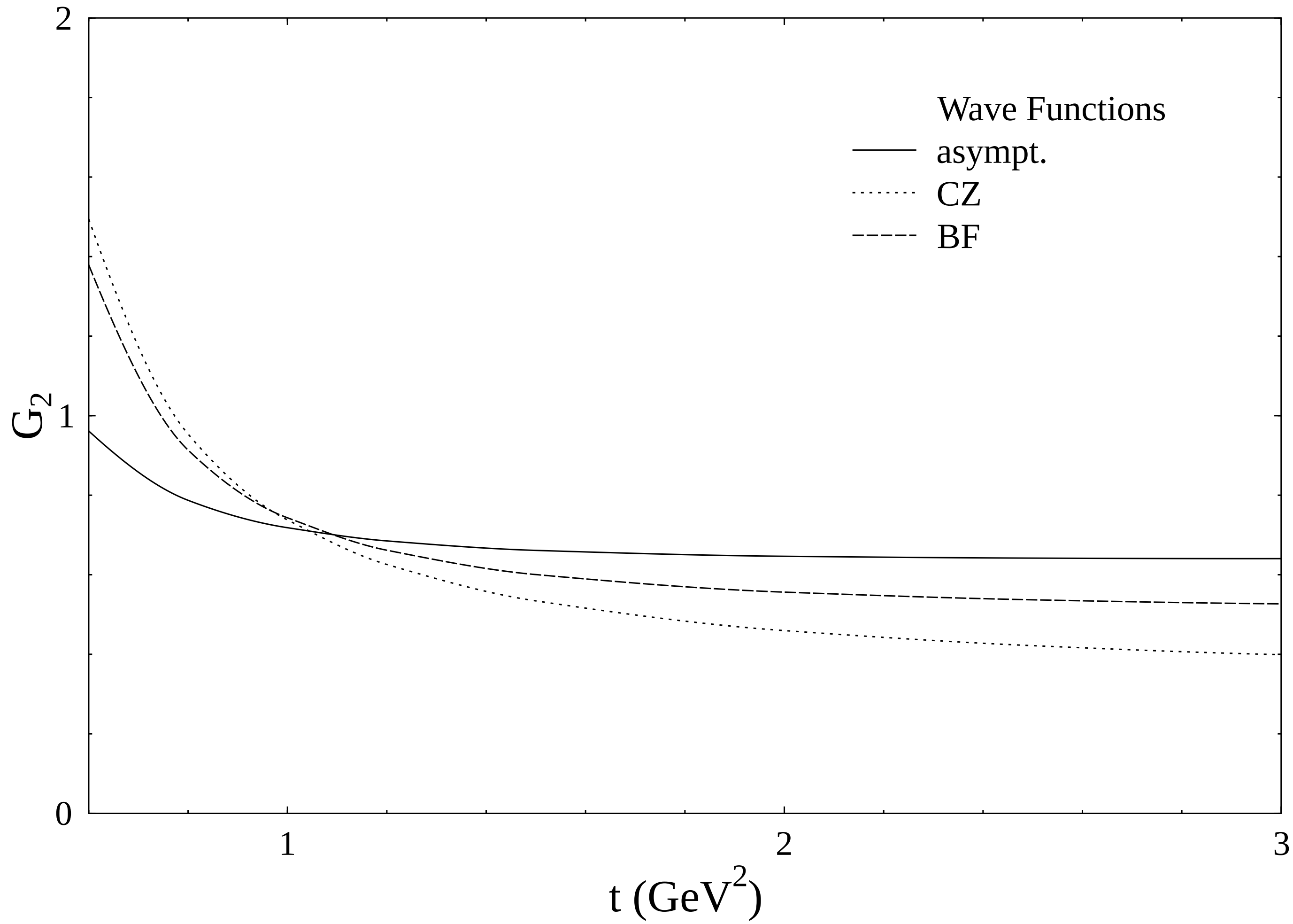

Fig.4. t-dependence of the sum rule for $G_2(Q^2)$, Q^2=2GeV^2.